The Paradox of the Pesticides

MSc Research Methods Presentation

Daniel Marshall

11th December 2023

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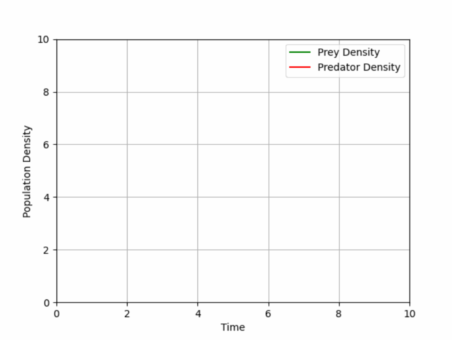

This is an example of the solution curves of the Lotka-Volterra system given parameters:
$\alpha = 3$
$\beta = 1$
$\gamma = 3$
$\delta = 1$
and initial conditions:
$x_0 = 5$
$y_0 = 5$

Plotting the solution curves in the x-y plane

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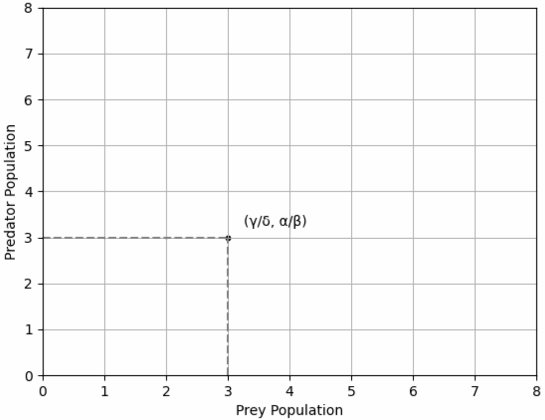
Let us take a look at a typical plot using the initial conditions discussed previously.



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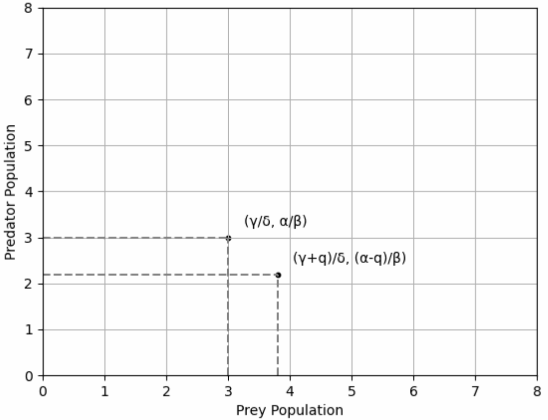
The animation below shows us what happens to the population both **before** and **after** adding pesticides.



The solution curves of the pesticide-affected system

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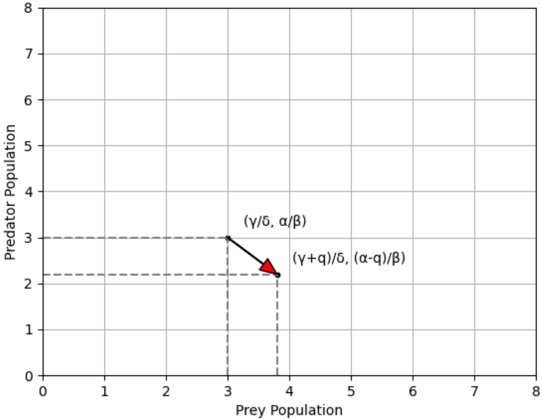

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We did this by observing what happened to the fixed points as we perturbed the system by a factor of q, accounting for the impact of pesticides on both population densities equally.

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