HOW DOES TEACHERS' AND STUDENTS' USE OF GEOGEBRA SUPPORT THE UNDERSTANDING OF GEOMETRY?

In this manuscript, we begin to evaluate the use of GeoGebra in aiding students' understanding of various geometrical concepts outlined in the UK GCSE Mathematics curriculum, providing both qualitative and quantitative data to justify our claims. Though the investigation was left inconclusive, due to the international COVID-19 pandemic, this script should provide a comprehensive insight on how the investigation was intended to be conducted. We further wanted to investigate to what extent mathematical applications were being used by teachers and students. In an ever-changing educational landscape, we were particularly interested in the number of teachers within our partner schools using technology in the classroom and, more specifically, which applications. We acknowledge the importance and benefit of independent study in mathematics and considered whether the introduction of the GeoGebra application to students would encourage a greater degree of self-learning through investigation of the application on mobile devices.

INTRODUCTION

The intention behind this action research project was to provide a solution to what we, as a department, have observed across resit groups for both GCSE and Functional Skills; namely, a drop in the reasoning and problem-solving ability of students. This was particularly noticeable in geometry and, from our limited experience with programs such as GeoGebra and Desmos, we postulated that such applications may be used to support students' understanding and promote scholarship.

The correlation between a learner's appropriately guided use of GeoGebra and their subsequent geometrical comprehension is well-documented (Bhagat & Chang, 2015; Diković, 2009; Kutluca, 2013). Diković investigated the efficacy of utilising GeoGebra to teach undergraduate students some of the most important theorems of calculus and linear algebra. The authors constructed various applets, illustrating complex concepts such as Cauchy's Definition of Continuity (Jourdain, 1913), to then be able to test students on what they have learnt from the aforementioned plug-ins. Both pre and post assessments, with a notable Cranbach-Alpha reliability of 0.784, were conducted to a total sample size of 31 Serbian students from the renowned Accredited Business School in Uzice, Serbia. Though the sample size was relatively small, the mean score increased by 28.73% between the mean scores of the pre-test to the post-test. One may interpret this as being a consequence of GeoGebra instruction, however the reader should remain sceptical of the cultural influences which were inevitably imposed on the results.

Bhagat and Chang conducted a similar pre and post assessment format to Diković, investigating the use of GeoGebra in the comprehension of important mathematical theorems. However, Bhagat and Chang assessed a sample of 50 Year 9 students from a government school in India on the fundamental circle theorems. The calculated effect size (η^2) was 0.213 between the two tests, which is significant (Cohen, 1988). Although this statistical analysis provides evidence for the basis of our manuscript, one must be aware of cultural and behavioural influences acting upon Bhagat and Chang's conclusion.

Kutluca conducted a rigorous investigation on the efficacy of GeoGebra on 42 Turkish Year 11 students' attainment score on the 25-item Van-Hiele Level of Geometrical Understanding Test (Usiskin, 1982). Although this is another small-scale study, the author calculated a Mann Whitney U-Test score of 82 ($\rho = 0.004$) between the pre and post test results of the experimental and control groups. One creative observation from Kutluca was that students began to peer-teach when experimenting with the pre-set GeoGebra applets. One may infer that the utilisation of intuitive GeoGebra applets could act as a vector for student engagement.

All of the aforementioned scripts provide a substantial basis on which to conduct our investigation. They have been chosen so as to impart the reader of the success of using GeoGebra to assist learners' comprehension of mathematics. Kutluca observed that GeoGebra may be utilised to promote engagement and collaborative teaching between students. Consequently, the inspiration for this manuscript was realised: Can educators utilise GeoGebra to improve the attainment of GCSE Mathematics resit students in a Further Education setting?

MOTIVATION

The motivation of this investigation is to evaluate and implement methods of improving the GCSE results of resit students nationally. The average National pass rate of June 2019 GCSE Mathematics resit examinees was 21.2%, with a negative 1.4%-point shift on the previous year (JCQ, 2019). We provide a relatively unique empathetic perspective on this issue, given the socio-economic background of our college. Wilberforce Sixth Form College is in an economically deprived area of Kingston Upon Hull - considered the 5th worst city in the United Kingdom for economic growth since 2009 (ONS, 2016). In order to illustrate this further, one should note that 50% of the college's 2020 student cohort are in the highest band of deprivation. Hence, the deduction can be made that the socio-economic disadvantages facing our students contribute to the relatively low mathematical achievement of our students. One may find this correlation to be well-documented (Sammons, 1995; Thomas et al., 1997; Tosto et al., 2016). This socio-economic issue manifests itself in the form of hereditary apathy and a heavy discouragement of offspring success (Heilman, 1929; Jerrim et al., 2015). This illustrates the necessity for creative methods of encouraging student engagement and conscientiousness toward mathematical success.

We wanted to use a form of graphical software, due to their ease of use and to potentially be a vector for flipped learning opportunities (Petrovici & Nemesu, 2015). After much discussion, we decided to use GeoGebra for its comprehensible design and vast adaptability (Reisa, 2010). The availability and convenience of GeoGebra overcomes the previously mentioned socio-economic issues (Edwards & Jones, 2006). GeoGebra integrates accessibility with interactivity, encouraging learners to discover mathematical axioms on their own, albeit with some guidance in the form of pre-constructed applets (Korenova, 2015). Thus, the motivation is well justified, and a method of such an investigation may now be discussed.

METHOD

The action research was to be split into several distinct stages, as outlined below:

Initially, we would gather data from a questionnaire using Google Forms in order to establish how confident teachers in our own department, and educators from our partner schools, already were in the use of technology within the classroom, and specifically to find out if they had any experience with GeoGebra. We selected the questions to be as broad as possible, enabling us to discover the usage - and the population's confidence with – various graphical software.

We then offered schools the opportunity to arrange a visit from a member of our own staff to demonstrate basic features of GeoGebra and share resources we planned to use in a later phase of the project.

The next stage of the action research would be to gather quantitative data to find out how students coped with some typical exam questions using their understanding of Geometry to solve multi-step problems. Examples of these can be found in Appendix B.

Subsequently, we then planned to deliver lessons for our students, using a number of resources gathered from the GeoGebra website as the main teaching element (external links in Appendix C). Teachers from our partner schools would also use these to deliver lessons but, instead of taking a prescriptive approach, we decided that it would be best to allow them to choose how to incorporate these into their teaching. This was done intentionally so that we could later gather anecdotal evidence on how effective different approaches had been, but also to try and ensure that teachers were able to fully engage with the project instead of feeling they had been forced into a particular teaching style. The resources selected could be used either as teacher led activities, or in some cases by allowing students to investigate properties. This notion was very much inspired by the aforementioned work of Kutluca.

A second assessment, with similar questions, would then be used to assess the same students after teaching had taken place. The motivation was that this would show some qualitative improvement in marks gained on each type of question. We wanted to eliminate as many variables as possible in this investigation. Thus, using analogous questions, which were testing identical skills, could accurately draw conclusions regarding students' geometrical knowledge acquisition.

Finally, we would use a final set of questionnaires combined with staff and student interviews to gather qualitative feedback and try to assess how successful GeoGebra had been in developing student understanding. The intention was that this would allow us to see how the project could be carried forward in order to further benefit both teachers and students.

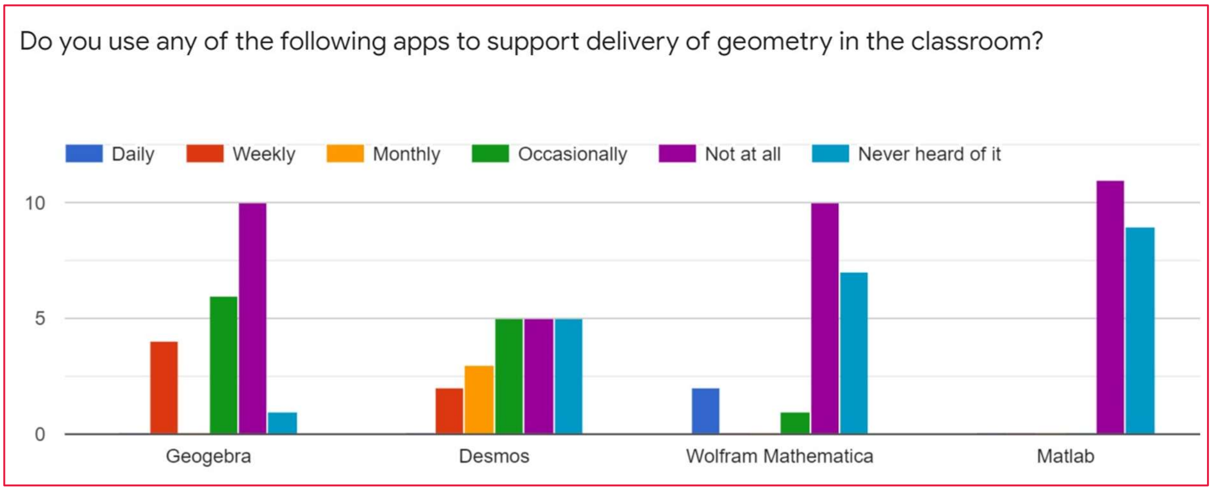
RESULTS

When itemising the results of this investigation, one must consider the unforeseen circumstances that were imposed upon us. Consequentially, we have the results of a modest number of assessment materials: The Initial Teacher Questionnaire and the Pre-Teaching Assessment. One is welcome to familiarise themselves with the aforementioned materials in both Appendix A and Appendix B of this document respectively.

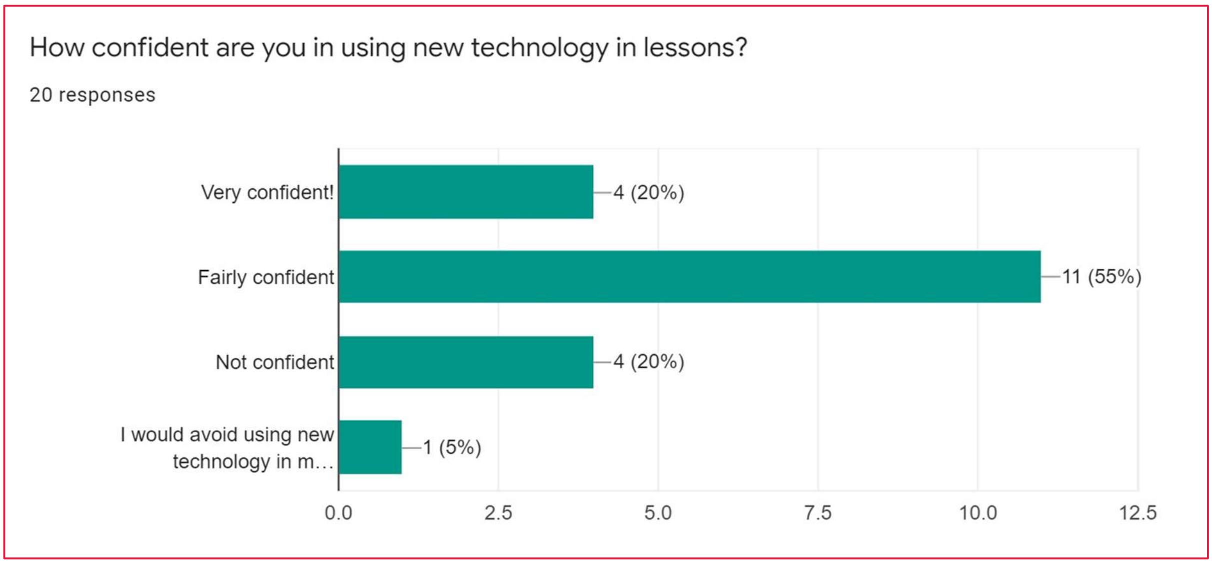
Initial Teacher Questionnaire

At the questionnaire stage of the research, we received 20 responses from teachers including our own department at Wilberforce and a number of staff from partner schools. These teachers all deliver the GCSE Mathematics curriculum, whether to students sitting it for the first time, or as a resit attempt.

Out of the four programmes identified, the graph below shows that Desmos and GeoGebra were more commonly used. However, at the time of answering, this was not a regular feature of teachers' delivery. MATLAB and Wolfram Mathematica were rarely used by teachers, and this makes sense given that these are typically used for higher level Mathematics. This result was reassuring in some respect as it reinforced the idea that Desmos and GeoGebra, with their focus on graphical interfaces and simple input options would be far more suitable for integrating into day to day lessons.

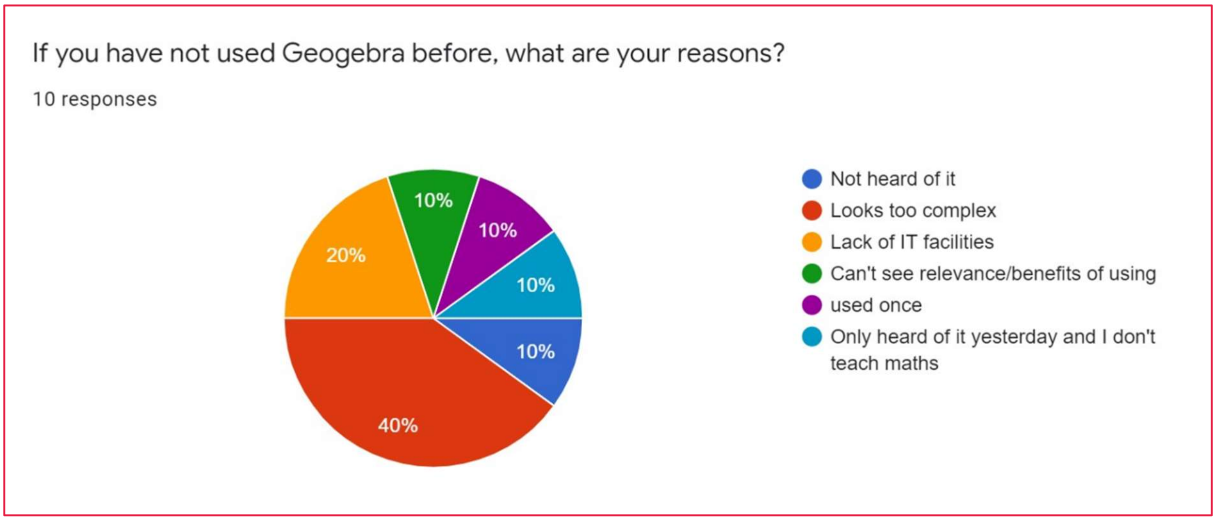


Our next questions were aimed at gaining an insight into teachers' confidence in technology and giving us the opportunity to identify the barriers of using software like GeoGebra in classroom activities.

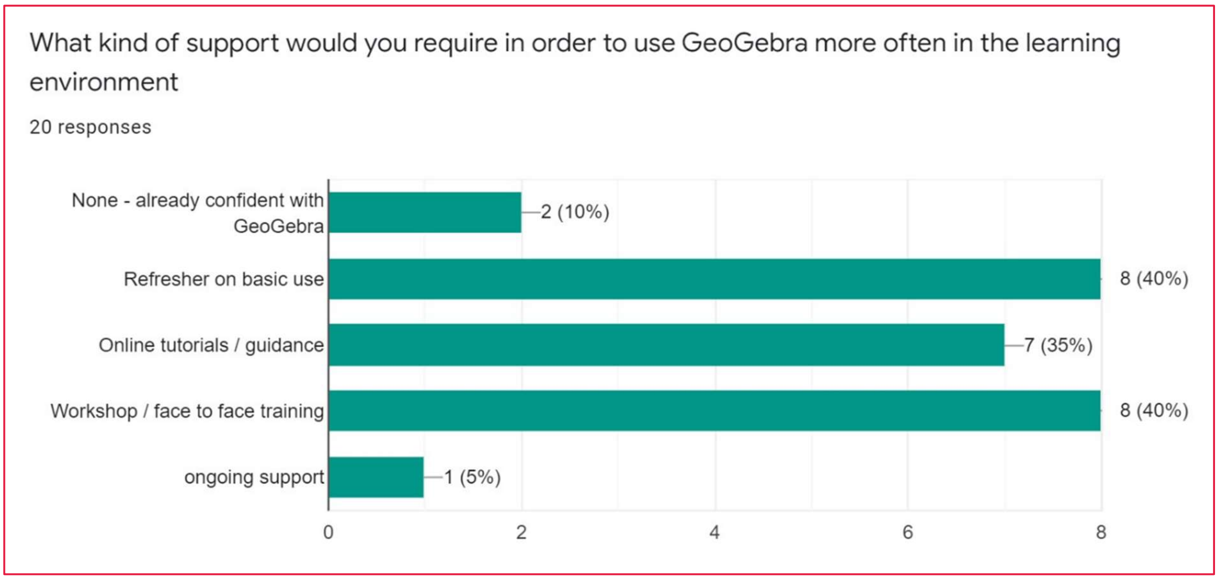


The results of this question indicate that, for the vast majority (75% of respondents) confidence in new technology is not an issue, which raises the question of why Mathematical software is not being used alongside more traditional teaching methods.

A follow up question revealed that for many teachers, GeoGebra itself was perceived as too complex, while the next most common response raised the issue of access to IT facilities. GeoGebra can also be used on mobile devices, which allows for the possibility of using students' own equipment, but this is highly dependent on schools individual policies, and as such we have not considered this for the purposes of our action research.



Finally, in order to move ahead with the project, we asked respondents what kind of support we could provide in the next stage of our action research.



The data here suggested that it would be beneficial to offer some face to face support with GeoGebra as a whole, as well as supporting partner schools to get to know the resources we would be asking them to incorporate into their lessons.

Pre-Teaching Geometry Assessment

Due to COVID-19, we were unable to access our other network partner schools to carry out the Pre-Teaching Geometry Assessment. An attempt to gather responses remotely proved futile and, as such, we only received a modest number of submissions (see Appendix B).

Typically, in the attempts received, students displayed little reasoning for geometry problems, and often did not show working methods. In the question on equations of straight lines, most students were able to identify intercepts, but struggled with gradients and made mistakes here.

Those students who attempted the transformation questions were confident in applying given transformations, but none managed to fully describe a transformation when given two images.

Finally, students who attempted the questions on angles in Polygons did better, though it is notable that some did not submit these at all. Students here relied on formulae, and while their work is well thought out, it is difficult to know whether this was done independently due to the nature of remote working.

CONCLUSION

It goes without saying that the COVID-19 pandemic has caused major upheaval to the education sector, but for this project the closure of schools prevented us from completing our research in the way we intended. The college closed the week before we were due to assess our students and we did not have chance to run lessons with the GeoGebra resources we had selected. As such, we have no way to measure how successful these resources would have been in improving students understanding of the topics.

We have provided a robust framework on how an ideal investigation into this topic should be conducted. Our survey questions were carefully chosen so that one may acknowledge the survey populations familiarity with GeoGebra and other graphical software platforms. We found that 40% of our educator population thought, upon seeing a GeoGebra demonstration, that the app was too complex. This was simultaneously worrying and intriguing, as it implies that an untapped method for potentially improving student engagement in GCSE mathematics lessons may exist. Moreover, a modest 10% of our teacher population stated that they were confident with GeoGebra: This suggests that in-house CPD, through staff validation, may potentially be a tactic to integrate GeoGebra into an educational institution.

This manuscript highlights the necessity for mathematics teachers of all levels to be aware of graphical software and its many capabilities. Consequently, CPD should be made available and distributable to accommodate this.

We still believe the project itself has merit and we hope to continue this into the next academic year, but there are now numerous other questions raised about the use of technology to support students understanding; namely how remote learning tools can be harnessed in the most effective way.

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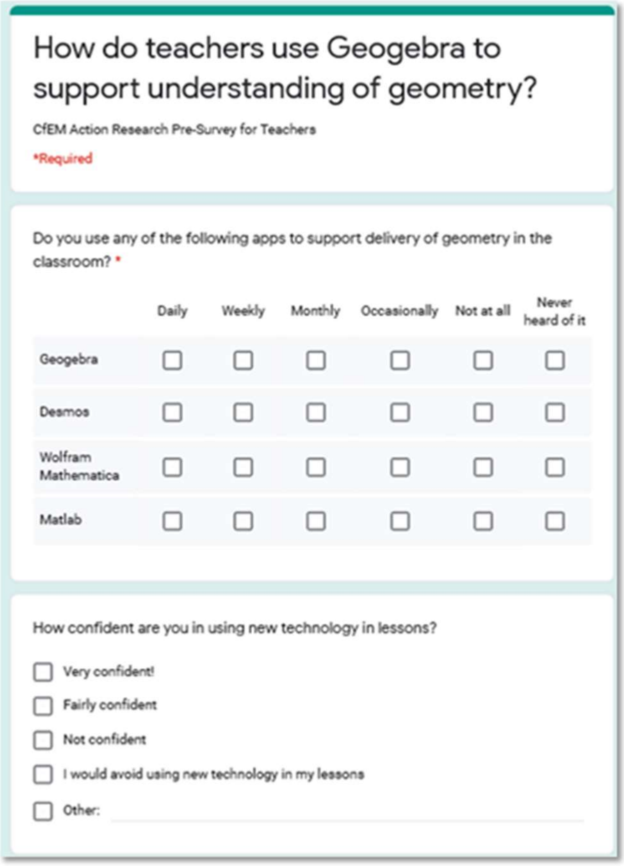
APPENDIX A: TEACHER QUESTIONNAIRES

In this appendix, we have attached the tools we have used to investigate the correlation between one's use of GeoGebra and their mathematical comprehension. We proposed an initial survey to find out how exposed the teachers at our partner schools are to graphical software, before honing our questioning in to specific GeoGebra usage.



How do teachers use Geogebra to support understanding of geometry?

CfEM Action Research Pre-Survey for Teachers

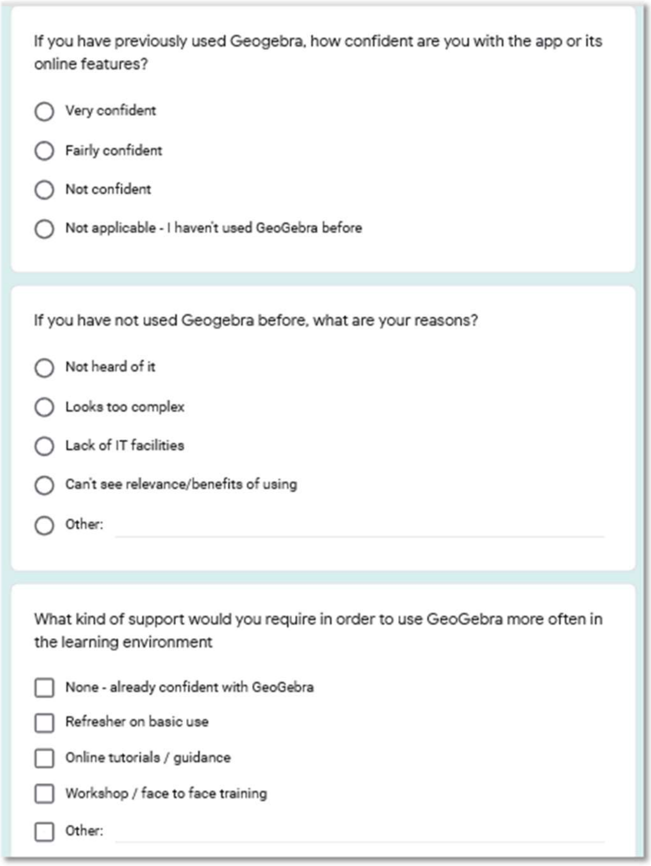
*Required

Do you use any of the following apps to support delivery of geometry in the classroom? *

	Daily	Weekly	Monthly	Occasionally	Not at all	Never heard of it
Geogebra	☐	☐	☐	☐	☐	☐
Desmos	☐	☐	☐	☐	☐	☐
Wolfram Mathematica	☐	☐	☐	☐	☐	☐
Matlab	☐	☐	☐	☐	☐	☐

How confident are you in using new technology in lessons?

- ☐ Very confident!
- ☐ Fairly confident
- ☐ Not confident
- ☐ I would avoid using new technology in my lessons
- ☐ Other: ______



If you have previously used Geogebra, how confident are you with the app or its online features?

- ○ Very confident
- ○ Fairly confident
- ○ Not confident
- ○ Not applicable - I haven't used GeoGebra before

If you have not used Geogebra before, what are your reasons?

- ○ Not heard of it
- ○ Looks too complex
- ○ Lack of IT facilities
- ○ Can't see relevance/benefits of using
- ○ Other: ______

What kind of support would you require in order to use GeoGebra more often in the learning environment

- ☐ None - already confident with GeoGebra
- ☐ Refresher on basic use
- ☐ Online tutorials / guidance
- ☐ Workshop / face to face training
- ☐ Other: ______

We then proposed a follow-up survey to evaluate how effective GeoGebra was from the educators' perspective (to be found on the next page of this manuscript). Due to the consequential circumstances of COVID-19, we were unable to gather any responses from this survey. For completeness, we have chosen to attach this survey so that one is able to evaluate the efficacy of our approach.

I have annotated each question of the post-GeoGebra questionnaire on the next page, so that one may evaluate our thought processes in developing an effective investigation into the efficacy of GeoGebra on mathematical comprehension. It was our aim, with this questionnaire, to discover whether such software is favoured by educators on its ease of use and versatility.

As an aside, we considered producing a survey for the students to complete subsequent to their exposure to GeoGebra. We ultimately decided against this, deducing that students' assessment scores – both before and after GeoGebra exposure – would suffice to justify our conclusions.

How do teachers use Geogebra to support understanding?

CfEM Action Research - post survey for teachers

How often do you now use Geogebra to support delivery in the classroom?

	Daily	Weekly	Monthly	Occasionally
Frequency	○	○	○	○

This first question was implemented to implicitly gauge whether teachers enjoyed using GeoGebra, as part of the investigation.

Are you planning to continue using Geogebra to deliver your lessons?

- ○ Yes
- ○ No
- ○ Other: ________

We wished to find out whether the introduction of GeoGebra inspired teachers to reconsider how they teach certain subjects within geometry.

Which aspects of Geogebra have been most useful in your teaching?

	Very useful	Somewhat useful	Not at all useful	Haven't used it
Graphing Calculator	○	○	○	○
Geometrical Constructions	○	○	○	○
Pre-made Activities/Resources	○	○	○	○
3D Calculator	○	○	○	○

One could perhaps use this as an avenue of investigation: What aspect of GeoGebra works best in the classroom?

How easy did you find it to integrate Geogebra into your geometry lessons?

	1	2	3	4	5	
Very easy	○	○	○	○	○	Too complicated!

This question was implemented to investigate how easy our educators found it to implement into lesson plans. The potential results of this question may be expanded to refine the provision of GeoGebra training.

How do you think that Geogebra has supported your students in developing their mathematical reasoning?

Your answer

Open questions allow for teachers to provide unique qualitative data

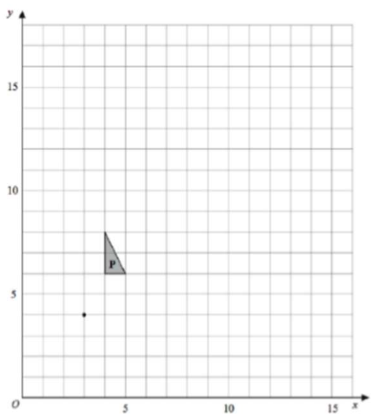
Are there any other comments that you would like to tell us, regarding your experience with Geogebra?

Your answer

APPENDIX B: STUDENT ASSESSMENTS

GeoGebra Action Research: Assessment 1

Question 1 - Transformations



(a) On the grid, enlarge triangle **P** with a scale factor 3 and centre (3, 4)
Label the new triangle **Q**.

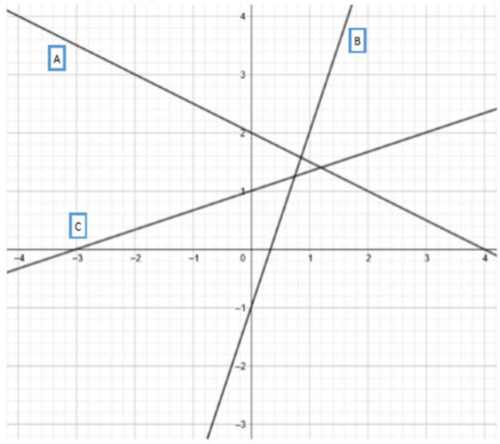
(b) On the grid, translate triangle **Q** by the vector $\begin{pmatrix} 4 \\ -8 \end{pmatrix}$
Label the new triangle **R**.

(c) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

(d) On the grid, reflect triangle **Q** in the line $x = 9$
Label the new triangle **S**.

GeoGebra Action Research: Assessment 1

Question 2 - Straight line graphs



Find the equation of each line:

A:

B:

C:

Decide whether each statement is "always true", "always false" or "sometimes true" explaining your decision in each case:

A: If two lines have different gradients they must intersect.

B: If a line intersects the y-axis at (0,-3) it intersects the *positive* x axis.

C: If a line has a gradient of 1 it makes an angle of 45^{0} with the x-axis.

D: If a line cuts the positive x-axis and the positive y-axis it has a negative gradient.

E: The line x=4 has a gradient of zero.

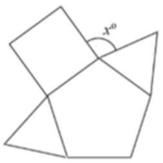
GeoGebra Action Research: Assessment 1

Question 3 - Angles in Polygons

The diagram shows a regular heptagon.

Find the value of x.
Show clear working out.

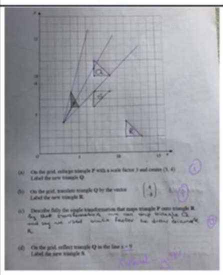
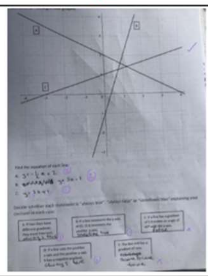
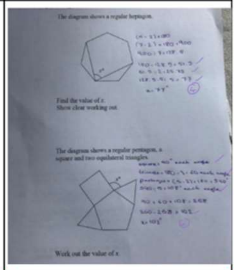
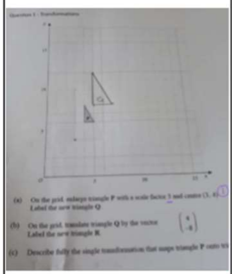
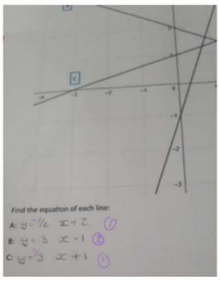
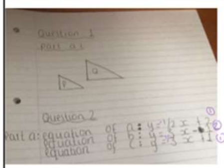
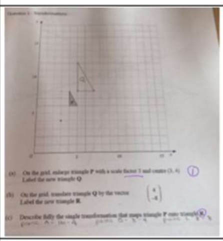
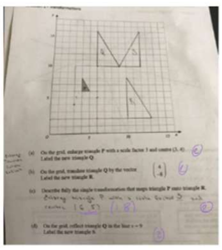
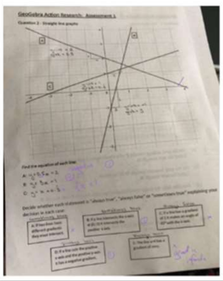
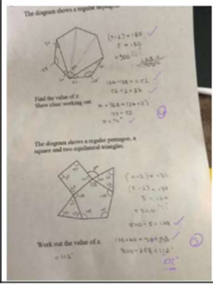
The diagram shows a regular pentagon, a square and two equilateral triangles.



Work out the value of x.

The Figure above shows the assessment we provided for the students before they were exposed to the pre-selected GeoGebra resources (see Appendix C). These questions were chosen so as to encompass the key concepts of elementary geometry.

The submissions we received from this first assessment can be viewed below:

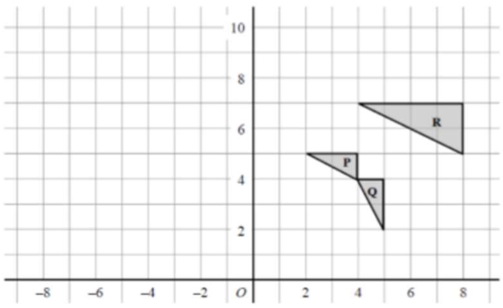
	Q1	Q2	Q3
Student 1			
Student 2			
Student 3			
Student 4			
Student 5			

One should notice that Student 1 and Student 5 are the exception and not the rule: That is to imply that levels of industriousness and engagement varied between our small sample size. It should be stressed that GeoGebra has the potential to improve the conscientiousness of unmotivated students. This should be an avenue of investigation for future researchers.

In preparation for carrying out a full-bodied investigation, we constructed an assessment for the students to complete after their exposure to GeoGebra. The assessment can be found in the figure below:

GeoGebra Action Research: Assessment 2

Question 1 - Transformations



(a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

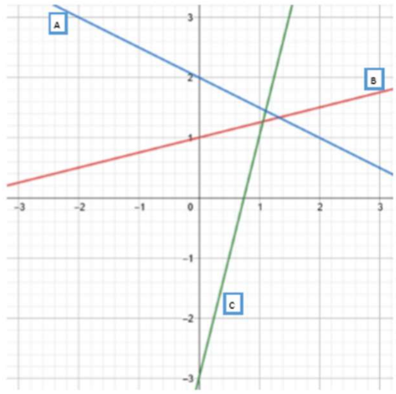
(b) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

(c) On the grid, reflect triangle **R** in the y-axis.
Label the new triangle **S**.

(d) On the grid, rotate triangle **P** 90° anti-clockwise about the point (–1, 3).
Label the new shape **T**.

GeoGebra Action Research: Assessment 2

Question 2 - Straight line graphs



Find the equation of each line:

A:

B:

C:

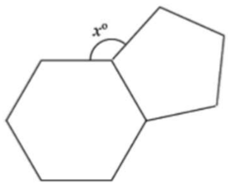
Decide if each statement below is "always true", "always false" or "sometimes true" explaining your decision for each:

A: Two lines with the same gradients will never intersect	B: If a line cuts the negative x-axis and the positive y-axis it's gradient must be positive	C: A line which intersects the x-axis at (3,0) will also intersect the positive y-axis
D: The line y = 3 has a gradient of zero	E: A line with a gradient of -2 must intersect the negative x-axis and the negative y-axis	

GeoGebra Action Research: Assessment 2

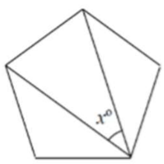
Question 3 - Angles in Polygons

The diagram shows a regular pentagon and a regular hexagon.



Work out the value of x.

The diagram shows a regular pentagon.



Find the value of x.
Show clear working out.

The questions were intentionally constructed to be similar to those of Assessment 1. This was done to eliminate as many controllable variables as possible: We did not want the objective difficulty of the questions to influence our conclusions. Though we were unable to utilise this second assessment, it should give an insight on how we wished to conduct our investigation.

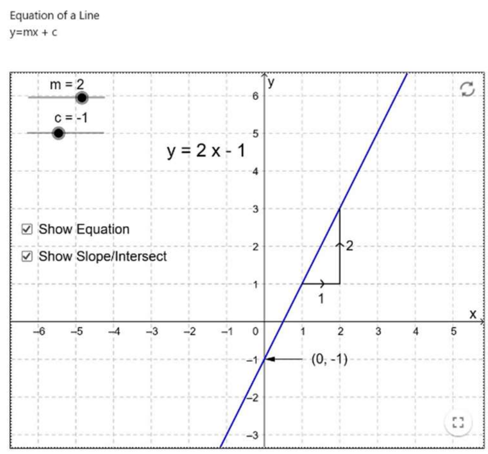
APPENDIX C: GEOGEBRA RESOURCES FOR TEACHING PHASE

The links below were intended to be used in the teaching stage of the project, with teachers choosing how best to utilise these depending on availability of IT facilities as well as their own personal teaching style. Screenshots have been included to give a basic idea, but the links included will allow readers to interact with these.

Equation of a Straight Line:

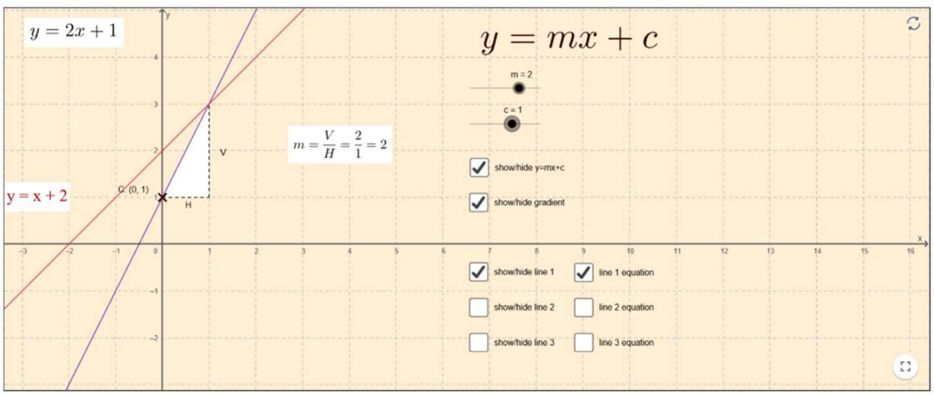
https://www.GeoGebra.org/m/zvxEFqyE

Applet can be used to investigate how changes to **m** and **c** affect the line visually and develop stronger links with the equation.



https://www.GeoGebra.org/m/BmwRPFeP

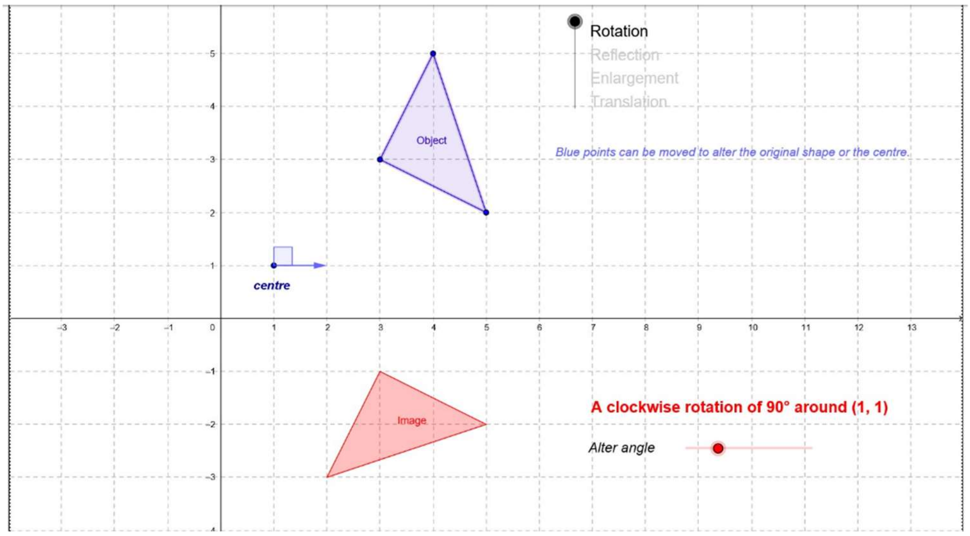
Match each line to examples, then find the equation using gradient triangles. Intended to reinforce learning from first link and check understanding of how to use gradient triangles to establish **m**.



Transformations Using Vectors:

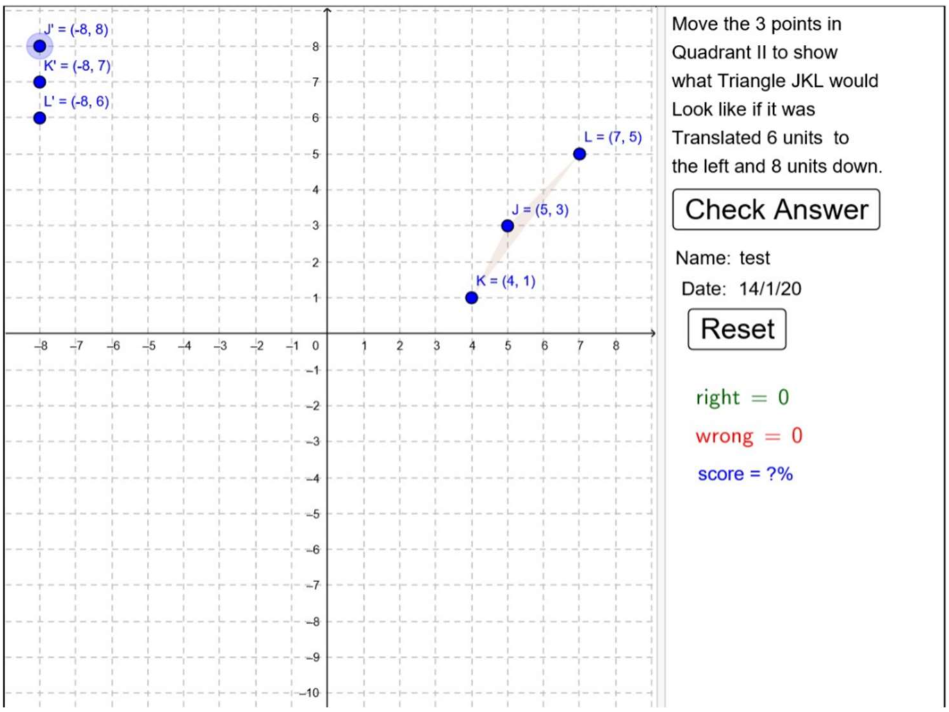
https://www.GeoGebra.org/m/hMNc4eES

Dynamic transformations tool to investigate reflection, rotation, enlargement and translation.



https://www.GeoGebra.org/m/GWwPSgpZ

Transformations game for students to apply transformations – self checking



Angles in Polygons and Triangles:

Note: Screenshots are not included for these activities as their dynamic nature is difficult to show in a static image https://www.GeoGebra.org/m/VafUetHY#material/mKzJCf5p

Exterior angles in Polygons - Students investigate and answer questions to check understanding

https://www.GeoGebra.org/m/VafUetHY#material/SwA5Q8jx

Interior and Exterior angles in quadrilaterals - Investigate then answer or discuss questions

https://www.GeoGebra.org/m/VafUetHY

Longer "Book" resource on angles and polygons which can be used as an extension or to follow up initial activities.